

NICA fixed target mode: soft jet studies in the relative 4-velocity space

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Abstract. Experimental results obtained by studying the properties of soft jets in the 4-velocity space at $\sqrt{s} \sim 2 - 20$ GeV are presented. The changes in the mean distance from the jet axis to the jet particles, the mean kinetic energy of these particles, and the cluster dimension in response to the growth of the collision energy are consistent with the assumption that quark degrees of freedom manifest themselves in processes of pion-jet production at intermediate energies. The energy at which quark degrees of freedom begin to manifest themselves experimentally in the production of soft pion jets is estimated for the first time. The estimated value of this energy is 2.8 ± 0.6 GeV. The suggestions are made for future investigations on NICA.

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1 Introduction

At the present time, the decision of the problem of confinement and the study of transition from meson-baryon degrees of freedom to quark-gluon ones is one of the most important (and, at the same time, most difficult) tasks of the world research program in the field of strong interactions. The nature of confinement of color degrees of freedom (quark and gluons) and, correspondingly, the possible phase transitions in strongly interacting matter is not completely clear so far. Observation of hadron jets at high energies is one of the most important and evident experimental manifestation of quark-gluon degrees of freedom. At present, an open question is where is the low boundary on energy starting with which the color degrees of freedom should be taken into account for the description of processes of multiparticle production. Obviously, the jet structure of events displays itself more clearly at high initial energies (\sqrt{s}) than at intermediate ones. But in spite of this feature it seems that the application of collective characteristics of multiparticle final state can be useful in the collision energy domain $\sqrt{s} \simeq 2 - 20$ GeV both for the study of transition from the predominance of meson-baryon degrees of freedom to the quark-gluon ones and for qualitative estimation of low boundary (for initial energy) experimental manifestation of quark degrees of freedom in soft jet production. In various fields of physics the onset of manifestation of new degrees of freedom and transition processes is accompanied by the presence of self-affine and fractal properties in collective effects. There-

fore, the precise measurements of collective and geometric (fractal-like) properties of soft pion jets in the NICA energy domain can give a new important information about hadronization mechanisms, behavior of quantum systems in the nonperturbative region and transition to manifestation of quark degrees of freedom in collective phenomena.

2 Method and variables

Traditional collective characteristics used for the study of the event shape [1] are not relativistically invariant. This introduces some additional kinematic uncertainties, for example, in the choice of the center-of-mass system for reactions with atomic nuclei. A relativistic-invariant method was proposed in [2] for studying collective effects in the case where particle-beam interaction with a target leads to the formation of a multiparticle final state in the reaction $b + t \rightarrow 1 + 2 + \dots$. Special features of this method in the case of the production of two jets were considered in detail in [3, 4, 5, 6, 7]. In that case, secondary particles refer to the region of target (beam) fragmentation if $X_t^k \geq (\leq) \tilde{X} \cap X_b^k \leq (\geq) \tilde{X}$, where $X_p^k = [m_k(U_k U_r)] [m_p(U_p U_r)]^{-1}$, $p, r = t, b$; and $p \neq r$; m_k is the mass of the k^{th} secondary particle; $m_{t/b}$ is the mass of the target/beam particles; $U_k = P_k/m_k$ is the 4-velocity; $k = t, b, 1, 2, \dots$; and $\tilde{X} = 0.1 - 0.2$ is some boundary value which is determined empirically. The basic quantities which the probability distributions (cross sections) depend upon are non-dimensional positive relativistic invariant quantities $b_{ik} = -(U_i - U_k)^2$, where $i, k = t, b, 1, 2, \dots$

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[2]. As seen the observables b_{ik} mean the squares of relative distances in the four-velocities space. The comparison of this method for distinguishing of some particle groups in the space of four-dimensional velocities with other present non-invariant (traditional) methods allows to name these separate groups as jets [8]. One of the most important observables of this approach is defined as $b_k = -(V - U_k)^2$, $k = t, b, 1, 2, \dots$, where $V = U_J/|U_J|$, $U_J = \sum_{i=1}^N U_i$ and N is the number of particles in the considered fragmentation region which satisfies all cuts and is involved in the analysis [2, 9]. The quantity b_k is the square of the distance of the k^{th} particle from the jet axis V in the space of U_k . The “temperature” defined as the mean kinetic energy of particles in the jet rest frame, $\langle T_k \rangle$, is estimated on the basis of the invariant functions $F(b_k)$ that, for pion jets, have the form [4] $F(b_k) = (\varepsilon/N) dN/db_k$, where $\varepsilon \equiv 4/[m_\pi^2 b_k \sqrt{1 + 4/b_k}]$, and which characterize the invariant cross section [9]. In [10], it was proposed to study the geometric properties of jets in the U_k space with the aid of the cluster dimension D defined on the basis of the relation between the number of particles in the jet being considered, $N(b_k)$, and its radius: $N(b_k) \propto b_k^{D/2}$. Non-integer value of the cluster dimension can be considered as characteristic signature of manifestation of fractal-like properties [11]. For most complex distributions the multifractal structure can appear and the cluster dimension be a function of jet radius for such case: $D = D(b_k)$. Thus D is the qualitative parameter reflecting the features of particle distribution in phase space. The set of observables $\mathcal{G} \equiv \{\mathcal{G}_i\}_{i=1}^3 = \{\langle b_k \rangle, \langle T_k \rangle, D\}$ characterizing the geometry and dynamics of the final-state production in the 4-velocity space is under consideration and is proposed for future investigation on NICA.

3 Recent results

Figure 1 gives the parameters from the set \mathcal{G} versus \sqrt{s} for various interactions at $\tilde{X} = 0.1$ (a, c, e) and $\tilde{X} = 0.2$ (b, d, f). The experimental-data array from [6, 7] is under study and various samples of these data are approximated by the functions

$$\mathcal{G}_i = a_1(\sqrt{s/s_0} - a_2)^{a_3}, \quad \sqrt{s/s_0} \geq a_2; \quad (1)$$

$$\mathcal{G}_i = a_1 + a_2 \ln(s/s_0). \quad (2)$$

where $i = 1 - 3$ and $s_0 = 1 \text{ GeV}^2$. One can see a qualitatively similar character of the energy dependence for all parameters from the set \mathcal{G} at the respective values of \tilde{X} . Figure 1 shows that, for any value of i , the dependencies $\mathcal{G}_i(\sqrt{s})$ exhibit a change in behavior in the region around $\sqrt{s} \simeq 3 \text{ GeV}$ for all interaction types, with the exception of hA , at any value of \tilde{X} , and this confirms the hypothesis put forth in [4] that dynamical interaction regimes undergo a change for $\sqrt{s} < 3 - 4 \text{ GeV}$.

In the region of $\sqrt{s} < 4 \text{ GeV}$, experimental dependencies $\mathcal{G}_i(\sqrt{s})$, $i = 1 - 3$, for all interaction types, with the exception of hA , are approximated by the function in (1).

The results of samples that combine the fragmentation regions are shown by dotted lines. One can see that the function in (1) agrees qualitatively with experimental data at all of the \tilde{X} values considered here. The parameter a_2 in (1) can be put in correspondence with the energy $\sqrt{s_c}$ at which quark–gluon degrees of freedom begin to manifest themselves in the production of soft pion jets. The values of $\sqrt{s_c}$ for the members of \mathcal{G} are given in the Table 1. The estimates of $\sqrt{s_c}$ for $\langle T_k \rangle$ and D agree well with each other but exceed the values of $\sqrt{s_c}$ obtained earlier in [5] on the basis of the $\langle b_k \rangle(\sqrt{s})$ dependence. This difference may be due both to physical reasons that lead to a sharper growth of $\langle T_k \rangle$ and D in relation to $\langle b_k \rangle$ and to a smaller size of the experimental-data samples in the case of the first two parameters. In the energy region being considered, an approximation by the function in (1) is also constructed for individual fragmentation regions. There is a substantial improvement of the quality of fits for any fragmentation region and any value of \tilde{X} ($\chi^2/\text{ndf} \sim 3 - 5$). Statistically acceptable values of χ^2/ndf could be obtained for 45% of the samples [6, 7]. Numerical values found for $\sqrt{s_c}$ from a fit to $\mathcal{G}_i(\sqrt{s})$ for $\sqrt{s} < 4 \text{ GeV}$ are summarized in the Table 1, the values corresponded to the fits of $\langle b_k \rangle$ energy dependence being taken from [5]. The values of $\sqrt{s_c}$ for the remaining two parameters from \mathcal{G} are presented in the first lines for $\langle T_k \rangle$ and D . Taking into account the sizes of samples of the available experimental data and the behavior of $\langle T_k \rangle(\sqrt{s})$ and $D(\sqrt{s})$, we can approximate the dependencies in question by the function in (1) for $\sqrt{s} > 2 \text{ GeV}$. A fit was constructed both for global samples and for the regions of target and beam fragmentation. For the first case, the fitted results are shown by the dashed curves in Figs. 1c – f. For $\langle T_k \rangle$, agreement between the approximation by the function in (1) and experimental data is only qualitative in the case of $\tilde{X} = 0.1$ (see Fig. 1c), since the fit quality is substantially poorer in that case than in the case of $\tilde{X} = 0.2$ (see Fig. 1d). For D , the fit quality is indicative of only qualitative agreement between the function in (1) and experimental data at either value of \tilde{X} (see Figs. 1e, f) [6, 7]. In the Table 1, the values obtained for $\sqrt{s_c}$ from a fit in the region of $\sqrt{s} > 2 \text{ GeV}$ for global samples and for individual samples in the target- and beam fragmentation regions are given in the second rows for $\langle T_k \rangle$ and D . The experimental results for the traditional definition of the thermal freeze-out temperature (T) obtained for pp collisions at $\sqrt{s} \approx 6 - 18 \text{ GeV}$ [12] agrees quite reasonably with $\langle T_k \rangle(\sqrt{s})$ at soft $\tilde{X} = 0.1$ (see Fig. 1c) in both the functional behavior and the magnitude. As consequence, the thermal freeze-out temperatures [12] are some smaller than the mean “temperatures” of pions in jets at hard cut $\tilde{X} = 0.2$ and close energies. It should be noted $T(\sqrt{s})$ calculated within the framework of the self-similarity approach assumes the sharp decrease at $\sqrt{s} < 5 \text{ GeV}$ and is almost flat at larger energies [13]. Such behavior of $T(\sqrt{s})$ agrees well with the features of $\langle T_k \rangle(\sqrt{s})$ (Fig. 1c, d) and confirms the hypothesis with regard to the changing of the dynamical regimes of multiparticle production at $\sqrt{s} \sim 5 \text{ GeV}$ [4].

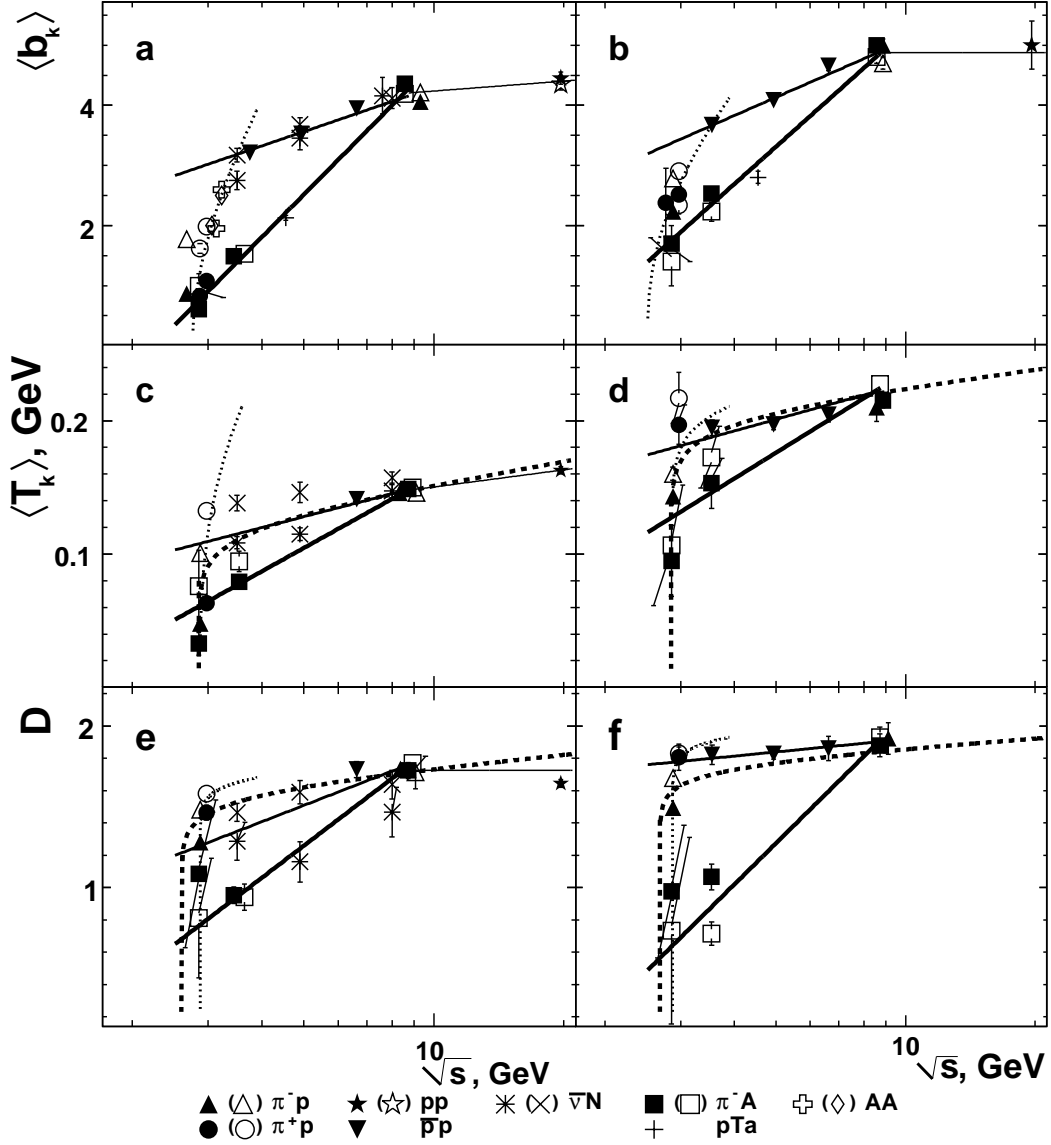


Fig. 1. Dependence of parameters from set \mathcal{G} on \sqrt{s} at $\tilde{X} = 0.1$ (a, c, e) and 0.2 (b, d, f) in the region of target (beam) fragmentation [7]. The dotted line corresponds to the approximation of data in the region of $\sqrt{s} < 4$ GeV by the function (1), while the dashed lines in Figs. c – f represent the approximation of all available experimental data by the function (1). The solid lines stand for the results obtained by fitting the logarithmic function (2) to a global sample of hh and $\bar{\nu}N$ reactions for $4 \leq \sqrt{s} < 9$ GeV (line with moderate thickness), data on hA reactions (thick line), and data on hh and hA interactions for $\sqrt{s} > 8$ GeV (thin line).

One can see that the values of $\sqrt{s_c}$ are reasonably consistent for the different \tilde{X} values and fragmentation regions. Thus, this investigation extended for the whole set \mathcal{G} renders the results obtained earlier in [4,5] more reliable and furnishes an additional argument in support of the hypothesis that a changeover of dynamical interaction regimes occurs because of the onset of the exper-

imental manifestation of quark degrees of freedom in the production of soft pion jets at $\sqrt{s} \sim 3$ GeV. This entails the respective transition from the description of the processes in question in terms of baryon–meson degrees of freedom to the use of quark–gluon degrees of freedom. The lower boundary of the energy corresponding to the onset of the experimental manifestation of quark degrees

Table 1. Values of $\sqrt{s_c}$ (in GeV units) at the boundary values of $\tilde{X} = 0.1$ and 0.2 [7]

| Parameter from set \mathcal{G} | Global sample | | Target fragmentation | | Beam fragmentation | |
|-------------------------------------|-------------------|-------------------|----------------------|-------------------|--------------------|-------------------|
| | 0.1 | 0.2 | 0.1 | 0.2 | 0.1 | 0.2 |
| $\langle b_k \rangle$ | 2.76 ± 0.01 | 2.51 ± 0.03 | 2.82 ± 0.02 | 2.46 ± 0.04 | 2.43 ± 0.04 | 2.5 ± 0.3 |
| $\langle T_k \rangle$ | 2.877 ± 0.001 | 2.865 ± 0.007 | 2.75 ± 0.04 | 2.865 ± 0.007 | 2.877 ± 0.001 | – |
| | 2.854 ± 0.001 | 2.854 ± 0.001 | 2.853 ± 0.001 | 2.854 ± 0.001 | 2.877 ± 0.001 | 2.854 ± 0.001 |
| D | 2.875 ± 0.001 | 2.875 ± 0.001 | 2.875 ± 0.004 | 2.877 ± 0.001 | 2.78 ± 0.04 | – |
| | 2.60 ± 0.03 | 2.68 ± 0.03 | 2.854 ± 0.002 | 2.858 ± 0.001 | 2.9 ± 0.4 | 2.874 ± 0.001 |

of freedom in the production of soft pion jets was estimated quantitatively for the first time. The interval in which the estimates of this parameter are contained and which is matched with the results for all of the collective features under consideration from \mathcal{G} (see Table 1) is $\sqrt{s_c} = 2.43 - 2.90$ GeV. Treating the set of estimates of $\sqrt{s_c}$ from the Table 1 as a sample of independent measurements, one can obtain $\langle \sqrt{s_c} \rangle = 2.78 \pm 0.14$ GeV. On the other hand, allowance for the amplitude of changes in $\sqrt{s_c}$ within one standard deviation (see Table 1) yields the interval $[(\sqrt{s_c})_{\min}; (\sqrt{s_c})_{\max}]$ for which the choice of midpoint and mean deviation makes it possible to obtain an estimate of $\langle \sqrt{s_c} \rangle = (2.8 \pm 0.6)$ GeV [6, 7]. A universal lower boundary for the manifestation of jet geometry for final states was qualitatively estimated in [14] at $\sqrt{s_l} \sim 3$ GeV for multiparticle-production processes. This value agrees with $\sqrt{s_c}$ with allowance for the errors. Possibly, the parameters $\sqrt{s_l}$ and $\sqrt{s_c}$ characterize the same physical effect – the onset of experimental manifestations of quark degrees of freedom in soft processes of the multiparticle production of secondaries and, hence, a manifestation of the jet structure of the event being considered. Thus, the results obtained by using traditional and four-dimensional collective variables agree reasonably and supplement each other.

Taking into account the results for $\langle b_k \rangle(\sqrt{s})$ in the region of $\sqrt{s} > 3.5$ GeV [4, 5] and the behavior of $\langle T_k \rangle(\sqrt{s})$ and $D(\sqrt{s})$ in the above range of \sqrt{s} , we approximated by the function (2) the samples taken for $\langle T_k \rangle$ and D and summed over the fragmentation regions. The numerical values of the fit parameters and the respective detailed discussion are presented in [6]. From Fig. 1, one can see that, for $3.5 < \sqrt{s} < 9$ GeV, the values of \mathcal{G}_i , $i = 1 - 3$, grow faster for hA reactions than for a nucleon target. Analyzing the results obtained for $\langle T_k \rangle$ and D in the range of $\sqrt{s} \sim 3 - 5$ GeV and taking into account large statistical errors in hA data, one can extend the conclusion concerning the effect of nuclear matter on the properties of soft pion jets [3] to the whole set \mathcal{G} . Approximations for $\langle T_k \rangle(\sqrt{s})$ and $D(\sqrt{s})$ in the region of $\sqrt{s} > 8$ GeV are possible only in the case of a soft limit on \tilde{X} (see Figs. 1c, e). In view of the scantiness of the available data, we cannot rule out definitively a weak logarithmic growth for D in accordance with Eq. (2). Thus, we see that, for all i , the dependencies $\mathcal{G}_i(\sqrt{s})$ admit a universal approximation in the form (2) for a broad class of interactions at

c.m. energies in the region of $\sqrt{s} > 3.5$ GeV and for both \tilde{X} values considered in the present study.

4 Further advancement

It is noteworthy, that agreement of the results of investigation of all parameters from the set \mathcal{G} with one another and the self-consistency of a global analysis of the properties of soft pion jets in U_k space renders the hypothesis of the onset of experimental manifestation of quark degrees of freedom in processes of the production of soft pion jets at $\sqrt{s} \sim 3$ GeV more plausible. However in order to test the above hypothesis it would be reasonable to perform additional investigations in the range of $\sqrt{s} \sim 2 - 20$ GeV. Furthermore the limited samples of experimental results for $\langle T_k \rangle$ and D and significant errors especially for $\sqrt{s} \sim 3$ GeV allow a semi-qualitative analysis only for these observables. Therefore additional high-statistic investigations on the NICA with various beams would be important for better understanding of the dynamic and geometric features of the production of soft pion jets.

The separation of various dynamics of jet production is a difficult task especially in the non-perturbative region of \sqrt{s} . The study of traditional collective observables at such \sqrt{s} allows the conclusion for event shape mostly [1]. Within the approach in question, the range $10^{-2} \leq b_{ik} \sim 1$ corresponds to the transition from the domain of dominance of meson-baryon degrees of freedom to the region where the internal structure of colliding particles and, as a consequence, quark-gluon degrees of freedom become essential in the processes of secondary particle jet production¹. The estimations for boundaries of various dynamic domains in terms of b_{ik} seem valid for b_k , i.e. when the jet axis corresponds to the “reference” i^{th} particle. That is why the distributions on b_k are usually studied. The approach under discussion allows the separation, at least, at qualitative level different dynamics of soft jet production. This advantage of the relativistically invariant method is important for intermediate \sqrt{s} namely. Therefore approaches for the analysis of jet production based on traditional collective characteristics and on the set \mathcal{G} are complementary to each other at intermediate \sqrt{s} . Complete investigation of jet production seems important especially at NICA for the energy region which corresponds to the

¹ One needs to note that the boundary values for b_{ik} indicated above are qualitative phenomenological estimations.

transition from the dominance of meson-baryon degrees of freedom to quark-gluon ones.

Another suggestion is the study of the behavior of α_S – the effective strong coupling constant – in the deeply non-perturbative region of \sqrt{s} . In the lowest order of the renormalization group equation (RGE) the following relation was derived $\alpha_S \propto 1/\ln b_{ik}$, where $i = t/b$ [9,15]. The possible exact relation between b_k and α_S requires an additional rigorous substantiation and careful derivation. Here one can note only that jets consist of particles with close masses (pions) in the present study and there is the relation $\langle b_k \rangle = 2(\langle M_J \rangle / \langle n_J \rangle m_h - 1)$ between the geometry quantity of jet in U_k space and mean invariant mass (M_J) of jet of hadrons with equal masses m_h [16, 17]. Here $\langle n_J \rangle$ is the mean multiplicity of particles inside the jet, the averaging is taken over particles in the event and over events in the sample in the l.h.s.; and over event ensemble in the r.h.s. The appropriate choice of the energy scale Q in the RGE is non-trivial for hadronic processes especially in the deeply non-perturbative region. By analogy with collider experiments [18,19] $\langle M_J \rangle$ is chosen as Q for some reaction at fixed \sqrt{s} within the framework of the method under study. Then based on the [20] one can derive the relation

$$\alpha_S = (b_0 \zeta)^{-1}, \quad \zeta \equiv 2 \ln[(0.5 \langle b_k \rangle + 1) \langle n_J \rangle m_h \Lambda^{-1}], \quad (3)$$

where $b_0 = (33 - 2n_f)/(12\pi)$ is the one-loop β -function coefficient, n_f is the number of quark flavors considered as light, Λ is the QCD parameter. The (3) demonstrates possible relation between soft jet characteristic $\langle b_k \rangle$ and strong coupling and gives indications on the additional advantage for study in non-perturbative regime at NICA. The estimations $\langle M_J \rangle \sim 1-2$ GeV obtained by the method under consideration for πp reactions at $\sqrt{s} \sim 3$ GeV [17] agree with the results at $\sqrt{s} \sim 9$ GeV [21]. Therefore the suggestion seems reasonable with regard to the validity of (3) in the NICA energy domain. It should be noted that there is one measurement of α_S in the deeply non-perturbative domain at $Q \sim 2$ GeV, namely from τ decay [20]. Thus the suggested equation (3) can provide the important estimation of α_S at $Q \sim \langle M_J \rangle$ on the order of few GeV and verifies the validity the theoretical curve $\alpha_S(Q)$ at low energy scales, i.e. it allows the investigation of one of the fundamental properties of QCD in a most difficult domain for theoretical description.

5 Summary

Summarizing the foregoing, we can draw the following conclusions.

The dependencies $\mathcal{G}_i(\sqrt{s})$, $i = 1 - 3$, exhibit qualitatively similar types of behavior, which admits a description in terms of power-law function for $\sqrt{s} < 4$ GeV and in terms of a logarithmic function for $\sqrt{s} > 3.5$ GeV. The behavior of $\mathcal{G}_i(\sqrt{s})$ for all i at $\sqrt{s} \sim 3$ GeV is likely to be due to the onset of an experimental manifestation of quark degrees of freedom in the production of soft pion jets and the respective transition from the description of the

processes in question in terms of baryon-meson degrees of freedom to the use of quark-gluon degrees of freedom. A lower limit on the energy at which quark degrees of freedom begin to manifest themselves in the production of soft pion jets was estimated quantitatively for the first time. The result is $\langle \sqrt{s_c} \rangle = (2.8 \pm 0.6)$ GeV. The effect of nuclear matter on the dynamical and geometric properties of soft pion jets in the 4-velocity space was found in the range of $\sqrt{s} \sim 3 - 5$ GeV.

The investigations of collective and geometric (fractal) properties in soft hadron and nuclear reactions at intermediate energies with high statistics at NICA can provide important progress for better a understanding of the deeply non-perturbative domain of strong interactions.

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